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The Effects of Traffic Load Variation on Measurement Accuracy

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A method is developed to quantify the effect of day-to-day variation in offered load on the accuracy of functions of traffic measurements. The method is applicable to any smooth function of the standard trunk-measurements—i.e., peg count, overflow, and usage. As an example, the accuracy of the trunks-required estimator for probability-engineered, full-access trunk groups is approximated. A sensitivity analysis shows that the major contributor to the variance of the estimator is day-to-day variation.

I. INTRODUCTION

Most of the traffic-engineering procedures in use in the Bell System require data collected over periods of up to several days. It has long been recognized that the daily offered loads estimated from trunk-group data show considerable variability even for data taken during the same hour of successive days.[1,2] This variability can cause significant differences between the observed blocking and the objective grade of service. It also can induce large fluctuations in the estimation of network requirements and must be accounted for in the traffic-engineering procedures. (For details on a model for this day-to-day load variation, see the Appendix.)

Two important applications that can be affected by day-to-day load variation are trunk servicing and trunk forecasting. The former is the use of traffic measurements to determine when trunk groups are significantly overloaded or underloaded. In this case, not allowing for day-to-day load variation can cause either repeated rearrangements (churning) or, more typically because of the emphasis on providing good service, overprovision of the traffic network. Trunk forecasting is the prediction of future network requirements. The accuracy of the forecast is strongly influenced by the day-to-day load variation. That accuracy, in turn, affects the procedures used to implement the forecast.

Most earlier work has attempted to quantify the effect of day-to-day load variation on averages of functions of traffic data.[2–6] Several trunk-engineering practices are now based on that work.[7] However, most of the previous studies that have examined statistical accuracy (as measured by variances) of functions of traffic measurements other than offered load have assumed that the true offered load (also called the source load) was a constant (e.g., Ref. 8). The purpose of this study is to provide an extension of the earlier work on traffic-measurement accuracy (which usually considered the stochastic nature of traffic and the effects of finite sampling) to include the effect of day-to-day load variation.

The most general work on the accuracy of single-hour measurements is that of Neal and Kuczura.[8] That work is used as a starting point for a more general model developed in Section II. The new model can be used to estimate the accuracy of any sufficiently differentiable function of the standard traffic measurements—i.e., peg count (number of arrivals), overflow, and usage. The model is used in Section III to approximate the standard deviation of estimates of the number of trunks required for probability-engineered groups. Section IV illustrates the application of the results of Section III to trunk servicing.

This paper uses concurrent work on mathematical models for day-to-day variation.[9] The reader should be familiar with that work, or for a short description of the main results, see the Appendix.

II. THEORETICAL RESULTS

For completeness, a brief review of notations and definitions is included here.[8] On each day, the measurements are taken over a time period denoted as $(0,t]$, with t usually taken to be one hour. The standard trunk-group measurements are:

(*i*) $A(t)$ is the measured number of arrivals (peg count) in $(0,t]$.

(*ii*) $0(t)$ is the measured number of overflows in $(0,t]$.

(*iii*) $L_d(t)$ is the measured usage based on a discrete scan [typically by a 100-second-scan traffic usage recorder (TUR)] of the number of busy trunks in $(0,t]$.*

It was found during this study that when day-to-day variation is included, the additional effect of the sampling errors in $L_d(t)$ is negligible for data from the message trunk network (see Section 3.3.5). Consequently, $L(t)$ will be used throughout the paper with the results being equally valid for the discrete, 100-second scan measurement, $L_d(t)$. The triple of measurements $(A(t),O(t),L(t))$ is taken for an interval of

* A continuous scan of the number of busy trunks (i.e., the total usage) is considered and is denoted by $L(t)$.

length t on each of n days. The total collection of data is described as $(A_i(t),O_i(t),L_i(t))$, $i = 1,\cdots,n$.

For the model assumed in this study, the trunk group contains c trunks, whose holding times are assumed to be from a negative exponential distribution with mean h. During the ith measurement interval, the arrival epochs form a renewal process with mean interarrival time λ_i^{-1}. The offered load during the ith interval is assumed to be constant and given by $a_i = \lambda_i h$, and the peakedness of the traffic is z.[1] The offered source loads $a_1,\cdots,a_n$ are assumed to be independent and identically distributed (iid) according to a specified probability distribution, Γ, which will be assumed to be a gamma distribution.[4] Note that because a_i and a_j, $i \neq j$, are independent, the processes associated with them are also independent.*

Any customer who arrives when there is an idle server will enter service immediately. Because this is a study for trunk groups with typically low blocking, a customer arriving to find all servers busy is assumed to depart and has no further effect on the system; i.e., customer retrials will be ignored.

2.1 The approximation

Let ξ_{ij}, $j = 1,2,3$, $i = 1,\cdots,n$ be the $3n$ random variables representing the data; i.e., $\xi_{i1} = A_i(t)/t$, $\xi_{i2} = O_i(t)/t$, and $\xi_{i3} = L_i(t)/t$. For a fixed i, each ξ_{ij}, $j = 1,2,3$ is a random variable whose parameters are functions of another random variable a_i. Denote the mean and conditional mean of ξ_{ij} by

$$\bar{\theta}_j = E[\xi_{ij}] = E[E(\xi_{ij}|a_i)],$$

and

$$\theta_j(a_i) = E[\xi_{ij}|a_i].$$

Then, setting $\underline{\xi} = (\xi_{11},\xi_{12},\xi_{13},\xi_{21},\cdots,\xi_{n3})$ implies that the mean of $\underline{\xi}$ is

$$\underline{\theta} = E(\underline{\xi}) = (\bar{\theta}_1,\bar{\theta}_2,\bar{\theta}_3,\bar{\theta}_1,\cdots,\bar{\theta}_3).$$

Now consider any differentiable function $g(x_{11},\cdots,x_{n3}) = g(\underline{x})$. Expand g in a Taylor series about $\underline{\theta}$ neglecting terms of order greater than one to get

$$g(\underline{\xi}) \approx g(\underline{\theta}) + \sum_{i=1}^{n}\sum_{j=1}^{3}\frac{\partial g(\underline{x})}{\partial x_{ij}}\bigg|_{\underline{x}=\underline{\theta}}(\xi_{ij} - \bar{\theta}_j). \tag{1}$$

* Studies have shown that the day-to-day variation in peakedness is small and is negligible for most network engineering applications. Recent studies have also shown that the effect of a systematic variation in load (e.g., as a function of the day of the week) is also negligible.[10] Finally, simulation data from this study have indicated that including the effect of calls with different exponential holding times on one group (e.g., effective and ineffective attempts) have little effect on the results.

The mean of $g(\xi)$ is approximated by

$$E[g(\xi)] \approx g(\underline{\theta}) + \sum_{i=1}^{n} \sum_{j=1}^{3} \frac{\partial g(\underline{x})}{\partial x_{ij}}\bigg|_{\underline{x}=\underline{\theta}} E(\xi_{ij} - \bar{\theta}_j)$$
$$= g(\underline{\theta}).$$

For the variance of $g(\xi)$,

$$E[g(\xi) - g(\underline{\theta})]^2 \approx E\left\{\left[\sum_{i=1}^{n}\sum_{j=1}^{3} \frac{\partial g(\underline{x})}{\partial x_{ij}}\bigg|_{\underline{x}=\underline{\theta}} (\xi_{ij} - \bar{\theta}_j)\right] \times \left[\sum_{k=1}^{n}\sum_{l=1}^{3} \frac{\partial g(\underline{x})}{\partial x_{kl}}\bigg|_{\underline{x}=\underline{\theta}} (\xi_{kl} - \bar{\theta}_l)\right]\right\}$$
$$= \sum_{i=1}^{n}\sum_{j=1}^{3} \frac{\partial g(\underline{x})}{\partial x_{ij}}\bigg|_{\underline{x}=\underline{\theta}} \sum_{k=1}^{n}\sum_{l=1}^{3} \frac{\partial g(\underline{x})}{\partial x_{kl}}\bigg|_{\underline{x}=\underline{\theta}} \times E[(\xi_{ij} - \bar{\theta}_j)(\xi_{kl} - \bar{\theta}_l)]. \quad (2)$$

Because a_i and a_k are independent for $k \neq i$, the expectation in (2) vanishes unless $k = i$, in which case it is[11]

$$E[(\xi_{ij} - \bar{\theta}_j)(\xi_{il} - \bar{\theta}_l)] = E\,\mathrm{Cov}(\xi_{ij},\xi_{il}|a_i) + \mathrm{Cov}(\theta_j(a_i),\theta_l(a_i)). \quad (3)$$

Because (3) does not depend on the subscript i (the a_i are iid), we may drop it and replace ξ_{ij} by $\xi_{\cdot j}$ and ξ_{il} by $\xi_{\cdot l}$ in the sequel. Substituting (3) into (2) provides the approximation for the variance*

$$E[g(\xi) - g(\underline{\theta})]^2 \approx \sum_{i=1}^{n}\sum_{j=1}^{3}\sum_{l=1}^{3} \frac{\partial g(\underline{x})}{\partial x_{ij}}\bigg|_{\underline{x}=\underline{\theta}} \frac{\partial g(\underline{x})}{\partial x_{il}}\bigg|_{\underline{x}=\underline{\theta}} \times [E\,\mathrm{Cov}(\xi_{\cdot j},\xi_{\cdot l}|a) + \mathrm{Cov}\,(\theta_j(a),\theta_l(a))]. \quad (4)$$

2.2 Computational considerations

The term in brackets in (4) is given by

$$\int [\mathrm{Cov}(\xi_{\cdot j},\xi_{\cdot l}|a) + (\theta_j(a) - \bar{\theta}_j)(\theta_l(a) - \bar{\theta}_l)]d\Gamma(a). \quad (5)$$

The functions in the integrand given in Ref. 8 are too complicated for the integral to be computed exactly, hence numerical quadrature is required.

In previous work several different quadrature schemes have been used on integrals similar to that in eq. (5). Because the functions are usually smooth, these schemes are generally successful. For this study, a compound 7th-order Newton-Cotes form was chosen.[12] The tails of the gamma distribution tend to zero sufficiently quickly that the infinite region of integration can be truncated to a finite region with no problem.

* When $g(\xi)$ includes the averaging of n days of data, each term $\partial g(\underline{x})/\partial x_{ij}$ contains the factor $1/n$, so that the variance of $g(\xi)$ is of the order of $(1/n)$.

III. EXAMPLE: VARIABILITY OF THE TRUNKS-REQUIRED ESTIMATE

The methods described in Section II were applied to the specific problem of computing the standard deviation, $\sigma(\hat{c})$, of the trunks-required estimate, $\hat{c}$, for probability engineered full-access trunk groups. The function g in this case is defined by a set of algorithms described in the Appendix. The partial derivatives of g are approximated by divided differences. Approximations were computed for several typical cases to cover a reasonable range of engineering interest. To test the accuracy of the approximations, they were compared with corresponding sample standard deviations from a simulation. These results are described in the following two sections. An application using these results to compute probability intervals for estimates of trunk required is illustrated in Section IV.

3.1 Basic calculations

For the main results, the existing trunk group size is fixed and the mean offered load $\bar{a}$, peakedness z, and levels of day-to-day variation are varied over the range of interest. Following Bell System practice, it is assumed that measurements are taken over a 20-day period (i.e., 20 independent one-hour measurement intervals) and that the trunk group is to be designed for an average-blocking objective of 0.01 (denoted $\bar{B}0.01$). It is also assumed that the calls have a mean holding time of 180 seconds. The sensitivity of the results to these assumptions are described in Section 3.3.

To validate the theoretical computations, sample variances from a simulation program were computed for each of several sets of input conditions (each variance was computed from a sample of size 50, which was large enough to give stable results and still be computationally feasible). For some input sets, the simulation runs were repeated to provide an indication of the variability of the estimated standard deviation. (An analytic approach would require computations of the 4th-order moments and cross-moments of the measurements and was not practical.)

3.2 Results

Results were computed for trunk groups ranging in size from 10 to 68 circuits. Illustrations of typical results are presented in Fig. 1 showing plots of $\sigma(\hat{c})$ as a function of input offered load and peakedness on trunk groups with 68 circuits. It has been observed in actual data that peakedness and level of day-to-day variation are correlated. Hence, combinations of peakedness and levels of day-to-day variation were selected to cover most values encountered in practice, with $z = 1$ and low variation selected to illustrate groups which first-routed traffic; $z = 4$ and medium,

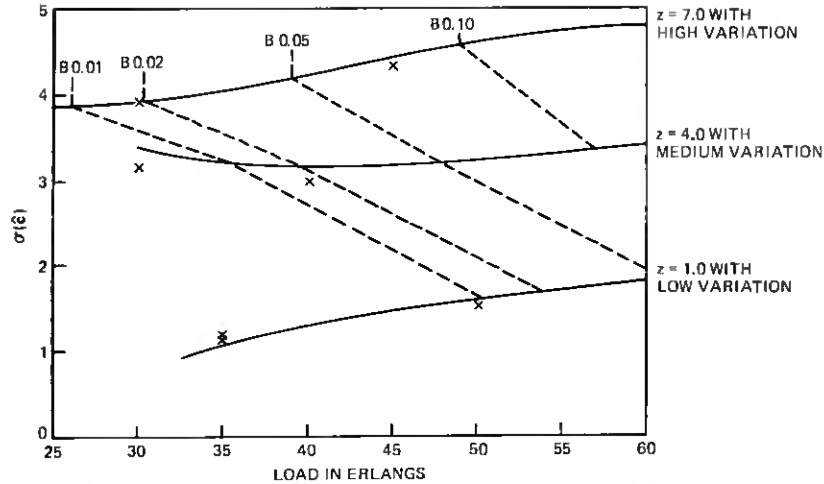

Fig. 1—Standard deviation of trunk estimate, $\sigma(\hat{c})$, vs offered load on a 68-trunk group.

and $z = 7$ and high were related to illustrate groups serving overflow traffic. In all cases, it was assumed that measurements are taken on a trunk group of the specified size, and the number of trunks needed to achieve an average blocking of 0.01 was estimated.

The input loads in general correspond to a range of blocking values from less than 0.01 to greater than 0.15. Except for $z = 1$, $\sigma(\hat{c})$ has a minimum at a load that corresponds to an observed blocking in the 0.02 to 0.03 range.* For lower blocking values and $z > 1$, $\sigma(\hat{c})$ increases as the load decreases because the coefficients of variation (standard-deviation-to-mean ratio) of the measurements, especially overflow, increase. As the load increases, $\sigma(\hat{c})$ also increases; however, the coefficient of variation of trunks required decreases slowly, probably because the coefficient of variation of the offered load decreases with increasing $\overline{a}$, causing the coefficients of variation of the measurements to decrease.[9]

3.3 Parameter sensitivity

The sensitivity of $\sigma(\hat{c})$ to the various parameters is illustrated in Figs. 2 and 3 and is described below.

3.3.1 Blocking objective

As illustrated in Fig. 2a, the first parameter tested was the design blocking. For the design range of 0.01 to 0.03,† there is very little change

* As discussed in the Appendix, when $z = 1$, the peakedness is not estimated, which causes the $\sigma(\hat{c})$ curve to have a different shape.

† The Bell System design objective is 0.01, but in some private networks and other administrations, higher values are used.

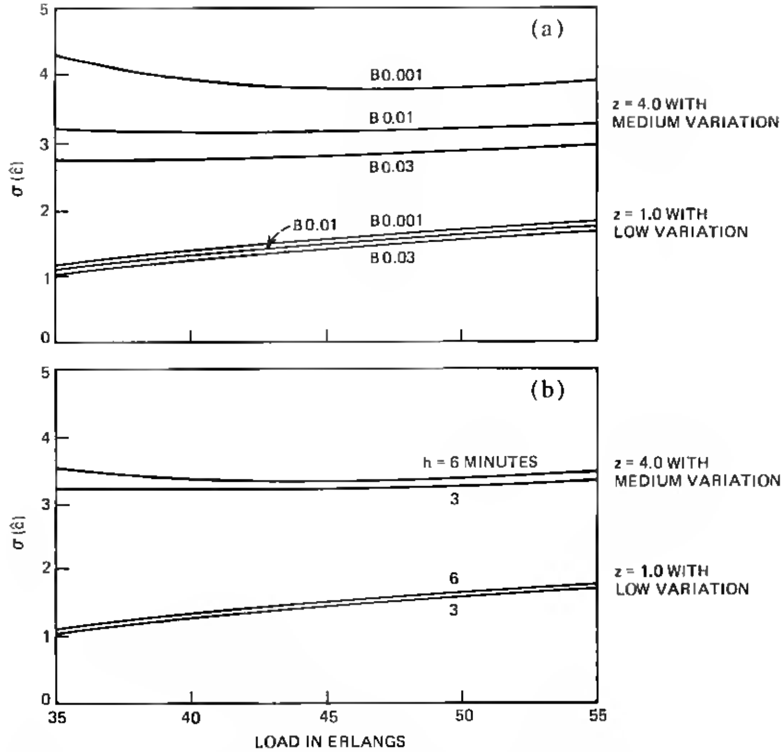

Fig. 2—Parameter sensitivity for a 68-trunk group. (a) Blocking objective. (b) Call-holding time.

in $\sigma(\hat{c})$ when $z = 1$ and low variation. For blocking in this range, the results for 0.01 can be used as an upper bound. Changes in the design blocking have more of an effect for more variable data, as illustrated by the data for $z = 4$ and medium variation.

3.3.2 Call-holding time

As the call-holding time h increases, the relative length of the one-hour measurement interval decreases. The result is a relative decrease in the amount of data available and a resultant increase in the standard deviation of the measurements.[8] However, for a fixed observed-load variance and for holding times in the range of 3 to 6 minutes, the effect is mostly offset by a decrease in the true day-to-day variation of the source load.[9] This is illustrated in Fig. 2b.

3.3.3 Level of day-to-day variation

The assumption to which the results are most sensitive is the level of day-to-day variation. The day-to-day variation of offered source-load is characterized by four levels, called *no*, *low*, *medium*, and *high*.

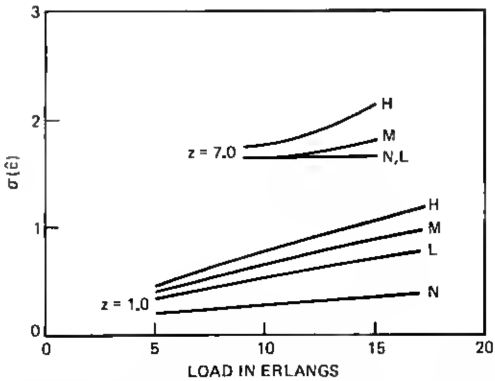

Fig. 3—Effect of level of day-to-day variation on a 10-trunk group.

Figure 3 provides a comparison of results for different levels of variation for a 10-trunk group. The labels, N, L, M, and H on the figure correspond to *no*, *low*, *medium*, and *high* variation, respectively.* The largest relative impact is for the case $z = 1$ when changing from *no* to *low* variation. In fact, the standard deviation of the trunk estimate doubles when such a change is made. Thus, inclusion of day-to-day variation in the model has a significant effect on the standard deviation of trunk estimates.

For large peakedness, shown by $z = 7$, as the load decreases, the curves coalesce. This rather unexpected behavior can be explained by the model for day-to-day variation (see Ref. 9). For groups with small loads, low day-to-day variation, and large z, the variance component due to sampling in a finite measurement-interval may be most or all of the total variance of the observed load. In this situation, the day-to-day component in the model of observed-load variance decreases to zero. For most practical applications, the large values of z are associated with medium or high levels of day-to-day variation, and this phenomenon does not occur.

3.3.4 Number of samples

As stated earlier, all of the plots are based on an average of 20 hours of data. If the number of hours, n, of available data is different from 20, the effect can be determined analytically from (4). The standard deviation for n hours is computed by multiplying the given results by the factor $(20/n)^{1/2}$.

3.3.5 Effect of usage sampling errors

The effects introduced by a discrete measurement of usage (TUR)

* These levels are defined in the Appendix.

Table I — Comparison of continuous and discrete usage measurement effects

c	a	z	*Level*	$\sigma(\hat{c})$ using L_d	$\sigma(\hat{c})$ using L
10	7.0	1.0	Low	0.4007	0.4372
10	7.0	4.0	Medium	1.012	1.043
40	30.0	4.0	Medium	2.138	2.174
40	30.0	7.0	High	2.949	2.999
68	40.0	7.0	High	4.172	4.211

were found to be negligible for the message-trunk network. Typical results of calculations using $L_d(t)$ and $L(t)$ are presented in Table I. The first four columns of Table I are the trunk group size, load, peakedness, and level of day-to-day variation. The next two columns are the approximations of the standard deviation of the trunk estimates with $L_d(t)$ and $L(t)$, respectively. The difference between the last two columns is negligible for traffic-engineering applications. Note that the relationship of the two columns is the opposite of what might be expected. This results from a bias in the asymptotic approximation for $\mathrm{Var}[L(t)]$ for the small loads included in the region of integration.[8]

IV. APPLICATION: PROBABILITY INTERVALS FOR TRUNK ESTIMATES

One of the first applications for the methods described in this paper was the development of probability intervals for trunk estimates. The intervals are used to determine if the estimated number of trunks required for a given circuit is (statistically) significantly different from the number presently in service. If the difference between the estimate and the current number is within the interval, then that difference is considered to be the result of the statistical nature of the data. Such a difference should not be the cause for action.

These intervals have application in two different areas of the trunk-engineering process. First, they provide an upper limit for the accuracy that can be attained by the trunk-forecasting process. Sources of error that have not been included here, such as wiring errors and load-projection errors, must increase the variability of the data. Second, they enable a trunk-servicer to evaluate the output of a mechanized trunk-servicing system and to determine if (and where) network rearrangements are necessary.

Data from the simulation described earlier indicate that tbe estimates of trunks required appear to have a normal probability distribution, with mean and variance computed as described in Section II. Using this in-

* This is not true when day-to-day variation is ignored.[8]

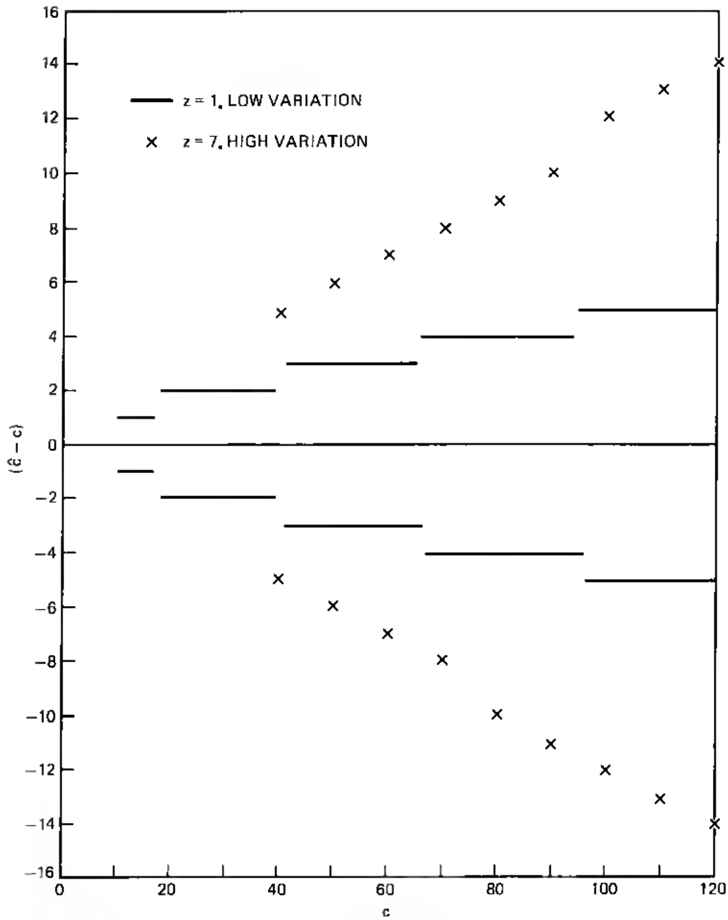

Fig. 4—Ninety-five-percent probability intervals for trunk estimates.

formation, the probability intervals for the trunk estimates have been constructed. Examples of such probability intervals are shown in Fig. 4. The solid line on Fig. 4 shows 95-percent intervals for trunk groups with $z = 1$ and low day-to-day variation. For example, for a correctly engineered 50-trunk group, 95 percent of the estimates of trunks required, based on traffic measurements, will lie between 47 and 53 trunks. The crosses on Fig. 4 show similar data for traffic with $z = 7$ and high variation.

The method described in Section II has also been successfully applied to approximate the variance of observed blocking for probability-engineered trunk groups. That result has been used to develop intervals of acceptable measured blocking for use in network servicing.

V. SUMMARY

A method for computing the effect of day-to-day variation in offered load on functions of traffic measurements has been presented. The method, which is applicable to any smooth function of the common traffic measurements, was used to compute the standard deviations of estimates of trunks required for probability-engineered trunk groups. In the associated sensitivity analysis, the daily variation in source load was identified as the significant contributor to the total variation of the estimate. In fact, day-to-day variation was so large that it was possible to neglect errors introduced by a discrete 100-second-scan measurement of the usage. (This extends a result derived analytically by Hayward for traffic with Poisson arrivals.[13])

The variability in trunk estimates depends very strongly on levels of peakedness and day-to-day variation. For first-route traffic and low day-to-day variation, probability intervals for trunk estimates grow slowly with c, while for higher levels of variation they expand rapidly. These results are presently being used to develop methods to assist trunk engineers in the forecasting and servicing of the traffic network.

APPENDIX

Details of the Computational Models

This appendix contains some of the engineering details necessary for the computations in Sections III and IV. The first section gives a brief description of the model for day-to-day load variation used in Section 3.1.[9] The second section discusses the conversion of traffic measurement data into estimates of the trunks required to meet an objective grade-of-service.

A.1 Model for day-to-day variation

Four levels of day-to-day load variation described as *no, low, medium,* and *high* are used for trunk engineering. For the latter three classes, the variance v of the measured (observed) offered loads is related to the mean offered load $\overline{a}$ by the formula

$$v = 0.13\,(\overline{a})^{\phi},$$

where $\phi = 1.5$, 1.7, or 1.84 for low, medium or high, respectively. The mean $\overline{a}$ is assumed to be constant during the measurement period (any variation of $\overline{a}$ during the measurement period will cause the estimated peakedness to be larger).

The variance of the observed loads is composed of two parts: the true source-load variance and the variance contributed by estimating the traffic parameters from data collected over a finite measurement interval.[9] The latter component is given by $2\overline{a}z/(t/h)$ where $\overline{a}$ is the mean

of the daily loads, z is the peakedness of the offered traffic, t is the length of the measurement interval, and h is the mean call holding time. Thus, by subtraction, the source-load variance is assumed to be

$$\mathrm{Var}(a) = \max\left\{0.13\overline{a}^{\phi} - \frac{2\overline{a}z}{(t/h)}, 0\right\}.$$

A more detailed discussion is given in Ref. 9.

A.2 Trunk-engineering process

The trunk-engineering process starts with an estimation of the traffic parameters obtained from trunk-group measurements. Time-consistent busy-hour measurements of the number of arrivals (peg count), the number of overflows, and usage are gathered for a period of several days (up to 20 business days when all data are available). They are then used to estimate the mean of the busy-hour loads and the peakedness of the offered traffic. The mean load is computed by averaging the hourly loads

$$\hat{a}_i = \frac{L_{di}(t)}{1 - \dfrac{O_i(t)}{A_i(t)}},$$

where $L_{di}(t)$, $O_i(t)$, and $A_i(t)$ are defined in Section II. The sample mean, $\hat{a} = 1/n\ \Sigma\ \hat{a}_i$ and sample variance $\hat{v} = 1/(n-1)\ \Sigma\ (\hat{a}_i - \hat{a})^2$ are computed next. In practice, the level of day-to-day load variation is selected by picking the value of ϕ as the one that provides the closest agreement between v and $\hat{v}$. For the computations here, ϕ was assigned by the program input. The next step in the traffic-engineering process is to apply a correction for the effect of retrials on $\hat{a}$ and $\overline{B} = 1/n\ \Sigma\ [O_i(t)/A_i(t)]$. (Because the region for the usual application of these results was for small blocking values, the retrial correction was not included in the analysis.)

The traffic peakedness is estimated by an iterative procedure. Given c, $\hat{a}$, and $\overline{B}$, a preliminary estimate of z is determined so that the theoretical blocking predicted by the equivalent random method matches the observed blocking.[1] The preliminary estimate of z is adjusted to correct for day-to-day variation using the procedure described in Ref. 7 to give the corrected estimate, $\hat{z}$, of the peakedness. In the case of trunk groups known to serve only first-offered traffic (i.e., none of the traffic has overflowed from some other group), the theoretical value of $z = 1$ is assumed and no estimation of peakedness is performed.

Once $\hat{a}$ and $\hat{z}$ have been determined, the number of trunks required to satisfy the engineering objective (usually $\overline{B}0.01$) can be determined from established trunk-capacity tables or appropriate computer algo-

rithms. These algorithms specify that any fractional trunk-requirement will be rounded up unless it is less than 0.3. This rounding rule induces the slight nonsymmetry seen in Fig. 4.

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